A Rigorous Method of Moments Solution for Curved Waveguide Bends and its Applications

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ABSTRACT

An accurate and computationally efficient method of moment solution together with a mode-matching technique for the analysis of curved planar waveguide bends is described. The method is applied to single and cascaded curved bends in rectangular waveguides, quantum waveguides and microstrips. The effect of the orientation of cascaded bends on the transmission properties is examined.

I. INTRODUCTION

The mathematical problem of obtaining the modal solutions of a curved waveguide bend has been treated by various researchers over many years [e.g., 1-8]. In 1948 Rice [1] formulated a matrix solution for curved rectangular waveguides by expanding the transverse eigensolutions in sine and cosine functions. He then used a limiting process to obtain approximate solutions for bends with large radius of curvature. Approximate modal solutions for a curved bend in a rectangular waveguide have been obtained by Lewin [2] by means of a perturbation analysis. Utilizing approximate formulas for Bessel functions, Cochran and Pecina [3] solved the appropriate characteristic equations for the propagating modes in a curved waveguide. In a recent publication, Accatino and Bertin [4] extended the approach of Cochran and Pecina by transforming the ill-conditioned characteristic equations for the evanescent modes in a curved rectangular waveguide into stable ones and calculated the reflection coefficient of curved E-plane bends in a rectangular waveguide.

Curved waveguide bends have also been analyzed in related areas such as acoustics and the recently emerged field of nanostructure physics [e.g. 5]. For example, Furnell and Bies [6] formulated a Ritz-Rayleigh variational procedure to analyze a curved bend in an acoustical duct. Sols and Macucci [7] and Lent [8], utilizing a finite-element method, have analyzed curved bends in quantum waveguide structures with ballistic quasi one-dimensional electron transport.

However, all the methods described above are either inaccurate (e.g., Rice [1]), tedious and/or computationally intensive. In this paper a rigorous and efficient method of moments analysis of curved waveguide bends is presented.

II. THEORETICAL FORMULATION

Figure 1 shows a general parallel-plate waveguide with electric or magnetic walls containing a curved bend discontinuity. The dielectric medium inside the waveguide may, in general, be an arbitrary (piece-wise continuous) function of the transverse direction. In this paper, however, only the case of a homogeneous dielectric medium is considered. The analysis of the general case will be published elsewhere.

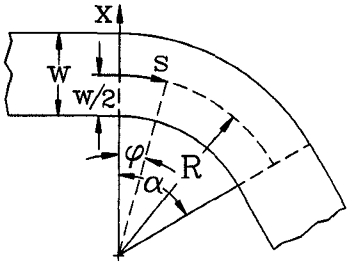

Fig. 1: Geometry of a curved bend in a parallel-plate waveguide.

The waveguide structure shown in Fig. 1 independently supports TE- and TM-modes. The solutions of the TE- and TM-modes in the parallel-plate waveguides are readily obtained from the scalar wave equation satisfied by the corresponding transverse electric or magnetic field component, respectively. The modal solutions in the curved region are conveniently found from the wave equation given in the curved coordinate system $(x,y,s{=}R\varphi)$ [2], [9] where R is the center radius of the bend (see Fig. 1). Inserting the modal solutions of the form

$$\psi(x,s) = f(x)\, e^{\pm j\beta s} \qquad (1)$$

into the wave equation leads to the eigenvalue equation

$$Lf = \left(u\frac{\partial}{\partial x}\left(u\frac{\partial}{\partial x}\right) + k^2u^2\right)f = R^2\beta^2 f\,, \qquad u = x+R \qquad (2)$$

with eigenvalues $\lambda_n = R^2\beta_n^2$ and eigenfunctions $f_n(x)$. The

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scalar quantity ψ corresponds to the transverse electric field component E_y for TE modes and to the transverse magnetic field component H_y for TM modes. Depending on the type of mode (TE or TM) and the type of plates (electric or magnetic walls), the boundary conditions are either of Dirichlet or Neumann type.

In order to convert the eigenvalue problem given in (2) into an equivalent matrix eigenvalue equation by means of the method of moments [10], the inner product

$$<g,h> = \int_{-w/2}^{w/2} \frac{1}{u} g(x)\, h(x)\, dx \tag{3}$$

with weighting function $1/u$ is defined. This choice of inner product makes the differential operator L together with the given boundary conditions self-adjoint. According to the method of moments the transverse field solution $f(x)$ is expanded into a set of complete basis functions $b_n(x)$ as

$$f(x) = \sum_{n=0}^{\infty} c_n b_n(x) \tag{4}$$

and the inner product of the eigenvalue equation with an appropriately selected complete set of testing functions $t_m(x)$ is taken. Here, Galerkin's procedure is employed where the transverse eigensolutions $\phi_n(x)$ of the corresponding parallel-plate waveguide are conveniently chosen as basis and testing functions ($b_n = t_n = \phi_n$). The resulting matrix eigenvalue equation reads:

$$\left(k^2 \boldsymbol{P} - \boldsymbol{S}\right) \boldsymbol{c} = R^2 \beta^2 \boldsymbol{Q} \boldsymbol{c} \tag{5a}$$

with

$$P_{mn} = \int_{-w/2}^{w/2} u\, \phi_m \phi_n \, dx \tag{5b}$$

$$S_{mn} = \int_{-w/2}^{w/2} u \frac{d\phi_m}{dx} \frac{d\phi_n}{dx} \, dx \tag{5c}$$

$$Q_{mn} = \int_{-w/2}^{w/2} \frac{1}{u} \phi_m \phi_n \, dx \tag{5d}$$

The integrals in (5d) can be efficiently computed by means of the Fast Cosine Transform (FCT) [11].

In order to obtain the scattering parameters of the curved bend structure, a mode-matching procedure [9] is employed at the interfaces between the straight and curved sections. Here, the mode-matching procedure is considerably simplified by the choice of basis and testing functions in the method of moments analysis and is simply given in terms of the matrices containing the expansion coefficients c_n for the transverse eigenfunctions in the curved region [11]. Having computed the scattering parameters of a single bend discontinuity, the scattering parameters of waveguide structures containing multiple bends are then obtained by applying a generalized scattering matrix technique [12], [13].

III. COMPUTATIONAL RESULTS

As part of the verification procedure for the method presented here, a curved bend in a quantum waveguide bounded by hard walls is considered. The analysis of the curved quantum waveguide is equivalent to that of a curved bend in a parallel-plate waveguide with electric walls and incident TE modes. The transmission coefficient [12], [13] as a function of the normalized phase constant is plotted in Fig. 2 and compared to the finite-element solution obtained by Lent [8].

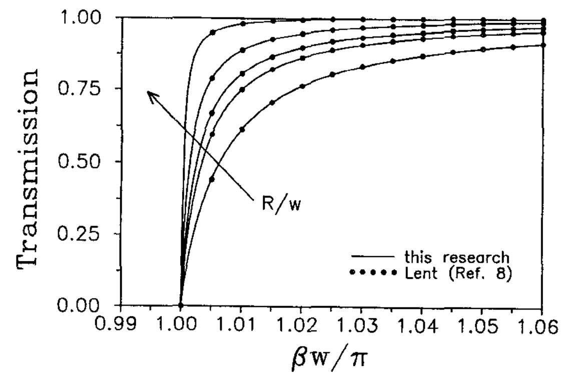
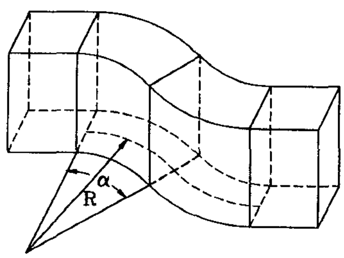
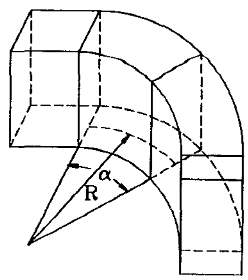


Fig. 2: Transmission coefficient as a function of the normalized phase constant for α=90° and R/w = 0.5, 0.65, 0.75, 1.0, and 2.0.

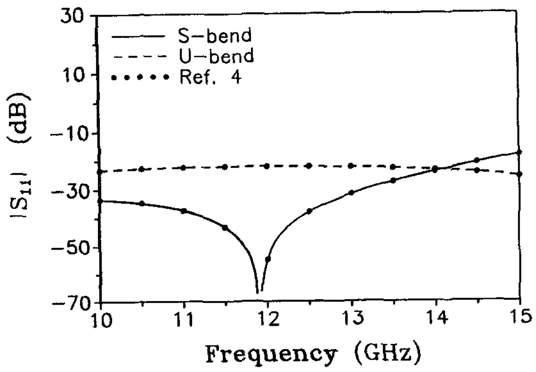
As a second accuracy test, the reflection coefficient of an S-shaped E-plane bend (Fig. 3a) and corresponding single E-plane bend (U-shaped bend) (Fig. 3b) in a rectangular waveguide has been calculated.



(a)



(b)

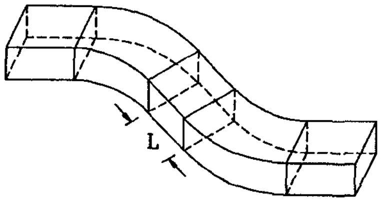


(c)

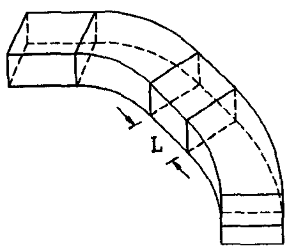
Fig. 3: Cascaded 45° E-plane bends with R=8mm in a WR-75 rectangular waveguide: (a) S-bend; (b) U-bend; (c) reflection coefficient.

As shown by Lewin [2], the characterization of rectangular waveguides containing E- and H-plane bends can be reduced to that of a corresponding parallel-plate waveguide configuration. Included in Fig. 3 are the results given by Accatino and Bertin [4]. This and the previous example clearly illustrate the accuracy of our method.

The results for S- and U-shaped bends shown in Fig. 3 indicate that it may be critical how the two bends are cascaded. In Fig. 4, cascaded H-plane bends are studied where a straight waveguide section of length L is inserted in between. With increasing length L the orientation of the cascaded bends becomes less significant and is nearly independent for lengths greater than the waveguide width over which the evanescent modes excited at the junctions are sufficiently attenuated.



(a)



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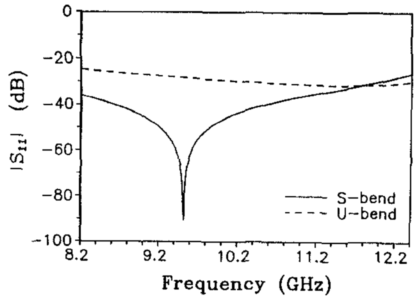

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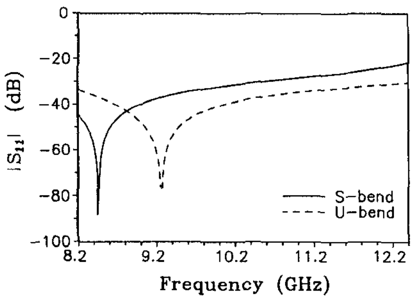
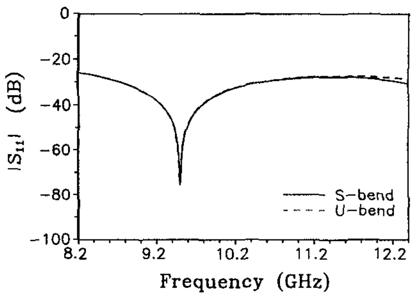


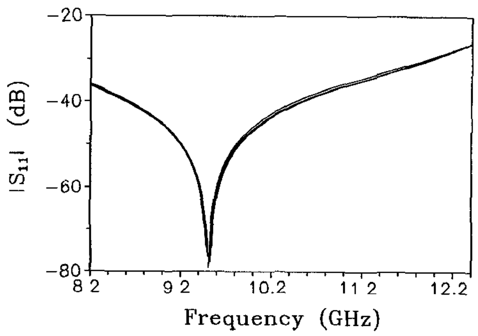
(d)



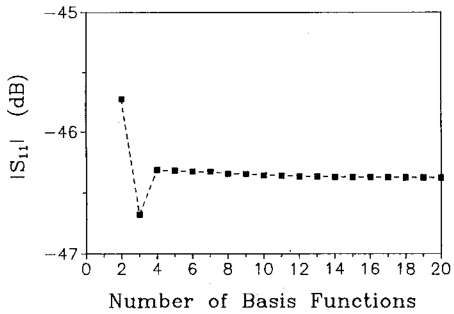
(e)

Fig. 4: Cascaded 30° H-plane bends with R=15mm in a WR-90 rectangular waveguide: (a) S-type bend; (b) U-type bend; reflection coefficient for (c) L=0, (d) L=5mm, and (e) L=25mm.

In order to appreciate the computational efficiency of the method, the reflection coefficient calculations of the S-shaped bend is shown in Fig. 5 for different numbers of basis functions retained in the method of moments procedure.



(a)



(b)

Fig. 5: Reflection coefficient of an S-shaped H-plane bend configuration (Fig. 4c) (a) for 2, 3, 4, 5, and 10 basis functions retained in the method of moments procedure, (b) as function of the number of basis function at f=10GHz.

In the second application of the method presented here, the double bend quantum waveguide configuration shown in the inset of Fig. 6 has been studied and compared to the corresponding structure with right-angle bends [12].

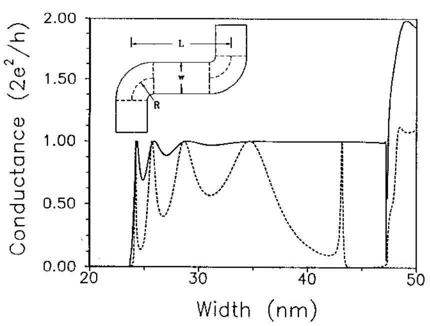


Fig. 6: Conductance of a curved double bend (solid line) and double right-angle bend (dashed line) in a quantum waveguide structure; L=150nm, R/w=0.52, m^*=0.067m_0, and E_f=10meV [12].

Figure 6 shows that the position of the conductance resonances [12] is nearly independent of the type of bend used. The amplitudes of the anti-resonances, however, are noticeably reduced with curved bends.

In the final application, the accuracy of our previously obtained perturbation solution for curved microstrip bends [9] is examined.

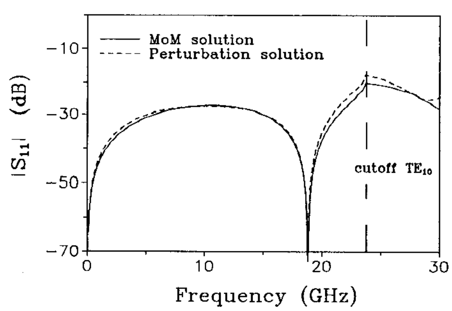


Fig. 7: Reflection coefficient for a curved microstrip bend with α=90°, R/w=2, w=1.2mm, h=0.635mm, and ε_r=9.8 [9].

It can be seen from Fig. 7 that the accuracy of the perturbation solution in this example is sufficient.

IV. SUMMARY

An accurate method for analyzing single and multiple curved bend discontinuities has been described and applications to varies waveguide types have been shown. The computed results for the scattering parameters converge very rapidly with increasing number of expansion functions and modes used in the method of moment and mode-matching analysis, respectively. As a rule, only a few expansion terms and modes need to be considered for accurate solutions so that the method described here is computationally efficient and can be implemented on an IBM AT or compatible desktop computer.

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